Systematic Study of Some Even-Even Nuclei $130 \leq A \leq 230$

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Abstract: The rotational bands of even-even isotopes are studied in the frame work of variable moment of inertia (VMI) and broken polynomial (BPM) models. The predicted results of the (B.P.) model are in close agreements with experimental data and other theoretical ones. By using broken polynomial, we will calculate the energies of yrast band from low up to high energy levels including the back bending/upending region. The theoretical calculations of the different models forms are analyzed and compared with the experimental data. It is found that excitation energies can be described successfully within this model.

Key words: rotational bands, variable moment of inertia (VIM), angular momentum

Introduction:

It was shown that the energy ratios, introduced by Mallman [1], have a good indicator in the study of heavy nuclei. The ratios $R\left(\frac{I}{2}\right)$ are defined as $R\left(\frac{I}{2}\right) = \frac{E(I)}{E(2)}$, where $E(I)$ is the excitation energy with angular momentum I. It is known that the ratio the ratio $R\left(\frac{4}{2}\right)$ is widely used as a good indicator of collectively .which is equal 2 in the vibrational limit, 3.33 in rotational limit, and values around 2.5 for $\gamma-$unstable nuclei. It was suggested by Cizewski [2] that plots of $R\left(\frac{6}{4}\right)$ versus $R\left(\frac{4}{2}\right)$ show a smooth systematic behavior, deviations from which indicate coexistence of collective and non-collective configurations. To study the behavior of the chosen nuclei, we use the ratio$R\left(\frac{I+2}{2}\right)$, $I = 2, 4, 6, \ldots$, which predict a distinctly behavior for all nuclei (Vibrational, rotational and $\gamma-$unstable limits).

Theory:

We know the excitation level energies of ground band $E(I)$ in rotational limit can be written as:

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$E(I) = AI(I+1)$, where I is the angular momentum and A is rotational constant. Then

$$R\left(\frac{I+2}{2}\right)_{Rot.} = \frac{(I+2)(I+3)}{I(I+1)} \ldots\ldots (1)$$

Also the vibrational bands in vibrational limit can be written as:

$$E(I) = BI \ldots\ldots (3)$$

Then

$$R\left(\frac{I+2}{2}\right)_{Vib.} = \frac{I+2}{I} \ldots\ldots (4)$$

It is clear that the above ratios $R\left(\frac{I+2}{2}\right)_{Rot.}$ and $R\left(\frac{I+2}{2}\right)_{Vib.}$ are decreasing with increasingI.

The relative ratio $r\left(\frac{I+2}{2}\right)$ is defined as [3]

$$r\left(\frac{I+2}{2}\right) = \frac{R\left(\frac{I+2}{2}\right)_{Exp.} - R\left(\frac{I+2}{2}\right)_{Vib.}}{R\left(\frac{I+2}{2}\right)_{Rot.} - R\left(\frac{I+2}{2}\right)_{Vib.}} \ldots\ldots (5)$$

where $R\left(\frac{I+2}{2}\right)_{Exp.}$ is the experimental ratio. It is clear that this ratio tends to zero for vibrational nucleus and tens to one for rotational nucleus, while takes values around 1.5 for γ-unstable nucleus. For other bands as octupole bands and intruder bands the band head energy must be considered, so the Equations (2), (4) are modified as:

$$R\left(\frac{I+2}{2}\right)_{Rot.} = \frac{(I+2)(I+3) - I_{bh}(I_{bh}+1)}{I(I+1) - I_{bh}(I_{bh}+1)} \ldots\ldots (6)$$

$$R\left(\frac{I+2}{2}\right)_{Vib.} = \frac{I+2-I_{bh}}{I-I_{bh}} \dots\dots (7)$$

$$R\left(\frac{I+2}{2}\right)_{Exp.} = \frac{E(I+2)-E(I_{bh})}{E(I)-E(I_{bh})} \dots\dots (8)$$

By using Equations (6), (7) and (8) we obtain the results of these kinds band.

1- Broken Polynomial Expressions:-

For ground-state bands of even-even nuclei, the γ-transition energy is characterized as E2 transition, where:

$$\Delta E(I) = E(I) - E(I-2) \dots\dots (0)$$

The angular velocity ω and the moment of inertia are defined by the following equations :(4), (5), (6), (7)

$$\hbar\omega_I = \frac{\Delta E(I)}{\Delta\sqrt{I(I+1)}}, \qquad \frac{\hbar^2}{2\theta_I} = \frac{\Delta E(I)}{\Delta[I(I+1)]} \dots\dots (9)$$

$$\omega_I\theta_I = \hbar J_I, \qquad J_I = \frac{1}{2}\left[\sqrt{I(I+1)} + \sqrt{(I-2)(I-1)}\right] \dots\dots (10)$$

One can reproduce both the known features of the spectra and the expected asymptotic behavior by broken expressions in terms of the angular momentum,

$$E(I) = \frac{P[I(I+1)]}{Q[I(I+1)]}, or \ \ \frac{\hbar\omega_I}{J_I} = \frac{\hbar^2}{\theta_I} = \frac{p\left(J_I{}^2\right)}{q\left(J_I{}^2\right)} \dots\dots (11)$$

where P, Q, p, q are assumed to be polynomials with few terms.

In systematic fits to the data with eq. (11) it is suitable to choose the coefficients in the numerator polynomial as varied parameters, while the denominator polynomial is used to achieve a prescribed asymptotic behavior.

The following expression is considered:

$$\left(\frac{J_I}{\hbar\omega_I}\right) = \frac{\theta_I}{\hbar^2} = \frac{a + bJ_I{}^2 + cJ_I{}^l}{1 + kJ_I{}^l} \dots\dots (12)$$

where k is a positive constant and l an even $interger \geq 4$. Eq. (12) is the simplest possible polynomial ratio which has three linear parameters $(a, b, and\ c)$ and fulfils the requirements for both low and high spins. We choose to extent the application of broken polynomial expression in terms of the angular momentum for describing the energy levels up to spin values $I > 30^+$, including the crossing in the yrast band. It should be interesting to apply the expression (12) along the states which are members of the crossing s-band. In our calculations, the expansion above the crossing region is taken as higher as $I_S = 34^+$, since, after I_{Cross} (1^{st} crossing),$E_\gamma^{g.b}(I) > E_\gamma^{s.b}(I)$, the switching from the $g.b$ to the $s.b$ is considered when $E_\gamma^{s.b}(I) < E_\gamma^{g.b}(I)$ for the same I.

2- The variable moment of inertia model (VMI):-

The energy levels of the ground state bands in even-even nuclei can be interpreted on the basis of a semi- classical model, in which the energy contains in addition to the usual rotational term, a potential energy term which depends on the difference of the moment of inertia θ_I (for the state of angular momentum I) from that of the ground state θ_0 [7, 8]. This model is called the variable moment of inertia model (VMI). In this model they assumed that there exists a variational expression for the energy in the form:

$$E_I = \frac{I(I+1)}{2\theta_I} + \frac{1}{2}C(\theta_I - \theta_0)^2 \dots\dots (13)$$

where C is the restoring force constant. For the super-band we allow the simplest possible form, i.e., a purely rotational band characterized by a constant moment of inertia [9, 10, and 11]:

$$E_S = E_0 + A[I(I+1) - K^2] \dots\dots (14)$$

In the above, E_0 is the band-head energy and A is the inverse of the constant moment of inertia.

RESULTS AND DISCUSSION:-

The energy levels of the even-even isotopes (subject of this paper) are calculated as predicted upon the bases of the two methods, as explained in last section.

Final VMI equation (13) as mentioned in the materials and methods section was used to calculated the energy levels (E_{VMI} -value) and was compared with the experimental data ($E_{Exp.}$-value) as mentioned in the experimental references [11].

The results obtained from this equation (13) were compared with the experimental data as given third column in table 1 and an insight concurrence was observed between the two values, and found good agreement E_{VMI} and $E_{Exp.}$.

The values of ground state moment of inertia (θ_0) parameter and stiffness parameter (C) is obtained by fitting (E_γ) energies by best fit method, and the values of the parameters obtained are listed in second and third columns in table 2. Also, three linear parameters ($a, b, and\ c$) and fulfils the requirements for both low and high spins and the values of these parameters obtained are listed in fourth, fifth and sixth columns in table 2.

In an attempt to obtain still better agreement with the experiment data, we have used equation (12).

It should be interesting to apply the expression (12) along the states which are members of the ground-state-band. We fitted equation (12) to the experimental data [11] for $2^+ \leq I \leq 28^+$ and calculated E(I) of the ground-state band to high spin for comparison with the experimental ground-state band cascades in nuclei; the results are given fourth column in table 1.

The results obtained are listed third and fourth columns in table 1 for even-even isotopes. The calculated energy levels are shown in table 1 as compared with the observed energy levels for the chosen isotopes. A good agreement is shown.

Table (1). Calculation in even-even nuclei using VMI model and Broken Polynomial Expressions (B. P.).

^{224}Th Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^+	0.0981	0.1065	0.0869
4^+	0.2841	0.3109	0.2748
6^+	0.5347	0.5807	0.5383
8^+	0.8339	0.9000	0.8528
10^+	1.1738	1.2598	1.2011
12^+	1.5498	1.6540	1.5746
14^+	1.9589	2.0783	1.9725
16^+	2.3980	2.3980	2.4015
18^+	2.8640	2.8640	2.8772
Mean deviation		0.0508	0.0139

^{226}Th Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^+	0.0722	0.0745	0.0686
4^+	0.2264	0.2347	0.2221
6^+	0.4473	0.4628	0.4473
8^+	0.7219	0.7455	0.7291
10^+	1.0403	1.0736	1.0536
12^+	1.3952	1.4406	1.4107
14^+	1.7815	1.8419	1.7943
16^+	2.1958	2.2740	2.2024
18^+	2.6351	2.6351	2.6370
20^+	3.0971	3.0971	3.1044
Mean deviation		0.0318	0.0072

^{228}Th Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0577	0.0585	0.0567
4^{+}	0.1868	0.1898	0.1851
6^{+}	0 .3717	0.3844	0.3774
8^{+}	0.6225	0.6333	0.6235
10^{+}	0.9118	0.9291	0.9140
12^{+}	1.2394	1.2659	1.2412
14^{+}	1.5995	1.6392	1.6001
16^{+}	1.9881	1.9880	1.9889
18^{+}	2.4079	2.4079	2.4091
Mean deviation		0.0158	0.0018

^{230}Th Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0532	0.0537	0.0522
4^{+}	0.1741	0.1758	0.1720
6^{+}	0.3566	0.3601	0.3547
8^{+}	0 .5941	0.5997	0.5943
10^{+}	0.8797	0.8884	0.8834
12^{+}	1.2078	1.2207	1.2151
14^{+}	1.5729	1.5924	1.5827
16^{+}	1.9715	1.9997	1.9809
18^{+}	2.3978	2.4396	2.4055
20^{+}	2.8500	2.9096	2.8535
22^{+}	3.3250	3.3249	3.3237
24^{+}	3.8120	3.8120	3.8159
Mean deviation		0.0165	0.0043

^{232}Th Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0493	0.0498	0.0484
4^{+}	0.1621	0.1633	0.1597
6^{+}	0.3332	0.3351	0.3301
8^{+}	0.5569	0.5592	0.5544
10^{+}	0.8270	0.8299	0.8267
12^{+}	1.1371	1.1421	1.1405
14^{+}	1.4828	1.4920	1.4901
16^{+}	1.8586	1.8759	1.8701
18^{+}	2.2629	2.2911	2.2765
20^{+}	2.6915	2.7351	2.7060
22^{+}	3.1442	3.2060	3.1566
24^{+}	3.6196	3.7019	3.6273
26^{+}	4.1162	4.2214	4.1183
28^{+}	4.6318	4.6318	4.6310
30^{+}	5.1620	5.1619	5.1680
Mean deviation		0.0252	0.0059

^{234}Th Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0495	0.0498	0.0493
4^{+}	0.1630	0.1642	0.1629
6^{+}	0.3365	0.3394	0.3368
8^{+}	0.5648	0.5709	0.5658
10^{+}	0.8430	0.8430	0.8430
12^{+}	1.1602	1.1602	1.1604
Mean deviation		0.0059	0.0003

^{230}U Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0517	0.0521	0.0516
4^{+}	0.1695	0.1710	0.1693
6^{+}	0.3471	0.3515	0.3473
8^{+}	0.5782	0.5876	0.5785
10^{+}	0.8564	0.8564	0.8562
12^{+}	1.1757	1.1757	1.1757
Mean deviation		0.0036	0.0001

^{232}U Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0475	0.0478	0.0473
4^{+}	0.1565	0.1575	0.1561
6^{+}	0.3226	0.3250	0.3225
8^{+}	0.5410	0.5452	0.5414
10^{+}	0.8058	0.8131	0.8070
12^{+}	1.1115	1.1244	1.1133
14^{+}	1.4537	1.4751	1.4554
16^{+}	1.8281	1.8618	1.8289
18^{+}	2.2315	2.2315	2.2310
20^{+}	2.6597	2.6596	2.6603
Mean deviation		0.0087	0.0007

^{234}U Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0434	0.0437	0.0434
4^{+}	0 1433	0.1441	0.1431
6^{+}	0.2960	0.2978	0.2959
8^{+}	0.4970	0.5004	0.4972
10^{+}	0.7412	0.7476	0.7418
12^{+}	1.0238	1.0355	1.0247
14^{+}	1.3408	1.3604	1.3413
16^{+}	1.6878	1.7194	1.6881
18^{+}	2.0630	2.0629	2.0628
20^{+}	2.4642	2.4642	2.4644
Mean deviation		0.0084	0.0003

^{236}U Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0452	0.0455	0.0451
4^{+}	0.1494	0.1501	0.1492
6^{+}	0.3097	0.3105	0.3096
8^{+}	0.5222	0.5227	0.5225
10^{+}	0.7823	0.7823	0.7831
12^{+}	1.0853	1.0854	1.0864
14^{+}	1.4263	1.4282	1.4274
16^{+}	1.8009	1.8076	1.8013
18^{+}	2.2039	2.2208	2.2038
20^{+}	2.6317	2.6655	2.6313
22^{+}	3.0812	3.1395	3.0810
24^{+}	3.5500	3.5499	3.5508
26^{+}	4.0390	4.0390	4.0393
Mean deviation		0.0099	0.0004

^{238}U Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0449	0.0451	0.0446
4^{+}	0.1483	0.1490	0.1476
6^{+}	0.3071	0.3081	0.3066
8^{+}	0.5181	0.5184	0.5178
10^{+}	0.7759	0.7756	0.7769
12^{+}	1.0767	1.0755	1.0790
14^{+}	1.4155	1.4145	1.4190
16^{+}	1.7884	1.7896	1.7921
18^{+}	2.1911	2.1979	2.1940
20^{+}	2.6191	2.6371	2.6206
22^{+}	3.0681	3.1051	3.0687
24^{+}	3.5353	3.6002	3.5356
26^{+}	4.0181	4.0180	4.0193
28^{+}	4.5170	4.5170	4.5183
Mean deviation		0.0102	0.0014

^{236}Pu Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0446	0.0448	0.0445
4^{+}	0.1474	0.1481	0.1473
6^{+}	0.3058	0.3073	0.3057
8^{+}	0.5157	0.5191	0.5161
10^{+}	0.7735	0.7796	0.7740
12^{+}	1.0743	1.0853	1.0746
14^{+}	1.4136	1.4136	1.4134
16^{+}	1.7860	1.7860	1.7861
Mean deviation		0.0041	0.0002

^{238}Pu Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0440	0.0442	0.0438
4^{+}	0.1459	0.1464	0.1452
6^{+}	0.3033	0.3048	0.3026
8^{+}	0.5135	0.5165	0.5133
10^{+}	0.7734	0.7786	0.7743
12^{+}	1.0801	1.0878	1.0820
14^{+}	1.4291	1.4411	1.4327
16^{+}	1.8185	1.8357	1.8229
18^{+}	2.2449	2.2689	2.2493
20^{+}	2.7057	2.7384	2.7090
22^{+}	3.1988	3.2420	3.1995
24^{+}	3.7208	3.7208	3.7190
26^{+}	4.2652	4.2652	4.2666
Mean deviation		0.0116	0.0018

^{240}Pu Nucleus			
I^{π}	$E_{Exp.}$	$E_{VMI.}$	$E_{B.P.}$
2^{+}	0.0428	0.0429	0.0426
4^{+}	0.1416	0.1422	0.1412
6^{+}	0.2943	0.2957	0.2936
8^{+}	0.4975	0.5004	0.4972
10^{+}	0.7478	0.7533	0.7482
12^{+}	1.0418	1.0510	1.0431
14^{+}	1.3756	1.3904	1.3780
16^{+}	1.7456	1.7687	1.7497
18^{+}	2.1520	2.1833	2.1553
20^{+}	2.5910	2.6318	2.5927
22^{+}	3.0610	3.1123	3.0608
24^{+}	3.5600	3.5600	3.5593
26^{+}	4.0880	4.0880	4.0890
Mean deviation		0.0148	0.0013

Table (2). Parameters for VMI model and Broken Polynomial Expressions (B. P.).

Parameters					
Nucleus	VMI Model		Broken Polynomial		
	C $(Mev)^{-3}$	θ_0 $(Mev)^{-1}$	a	b	c
^{224}Th	5.856855E-04	25.20034	68.2096	53.64423	-9.101096
^{226}Th	7.124271E-04	39.00757	86.80921	37.53948	-4.334693
^{228}Th	7.992510E-04	50.51387	105.2411	31.87792	-3.865061
^{230}Th	1.000366E-03	55.36955	114.5581	19.68331	-1.057857
^{232}Th	8.775768E-04	59.73208	123.6083	18.53043	-0.859742
^{234}Th	1.387866E-03	59.93797	121.2298	17.45467	0.216425
^{230}U	1.100121E-03	57.16476	115.6766	27.90818	-4.899238
^{232}U	1.046928E-03	62.28716	126.3552	19.86558	-1.395329
^{234}U	8.877636E-04	68.20182	137.8744	20.83973	-1.583053
^{236}U	1.119884E-03	65.62202	132.6849	15.03840	-0.511323
^{238}U	1.061694E-03	66.07312	134.1782	14.21247	-0.370761
^{236}Pu	1.397435E-03	66.71944	134.4021	15.07457	-0.736264
^{238}Pu	1.791186E-03	67.68134	136.8047	10.12822	-0.387009
^{240}Pu	1.451718E-03	69.60776	140.5333	12.67426	-0.718935

Table (3). The values of relative ratio for rotational, vibrational and gamma unstable nuclei.

I	Relative ratio $r(I+2/I)$									
	Rotational nuclei			Vibrational nuclei			$\gamma-$ unstable nuclei			
	Dy^{162}	TH^{232}	U^{238}	Dy^{154}	Gd^{152}	Yb^{158}	Pt^{182}	Pt^{184}	Pt^{186}	Pt^{196}
2	0.970277	0.962882	0.977625	0.174387	0.145608	0.247140	0.531960	0.503519	0.419452	0.349541
4	0.941174	0.925447	0.950375	0.231147	0.207815	0.303130	0.579441	0.547346	0.483065	0.402435
6	0.908802	0.887343	0.927413	0.249111	0.235945	0.328515	0.591652	0.545546	0.513210	0.376474
8	0.873545	0.846022	0.891314	0.248179	0.241192	0.325898	0.574851	0.490244	0.481374	0.281375
10	0.842642	0.801945	0.860194	0.254062	0.245504		0.551622	0.416666	0.444971	
12	0.801704	0.765249	0.824563	0.256962	0.260408			0.392776		
14	0.770223	0.725696	0.791328		0.269372					
16	0.721480	0.699111	0.756864							
18		0.669395	0.720121							
20		0.650964	0.681859							
22		0.635556	0.646916							
24		0.621510	0.614217							
26		0.605953	0.592190							

28		0.582487								

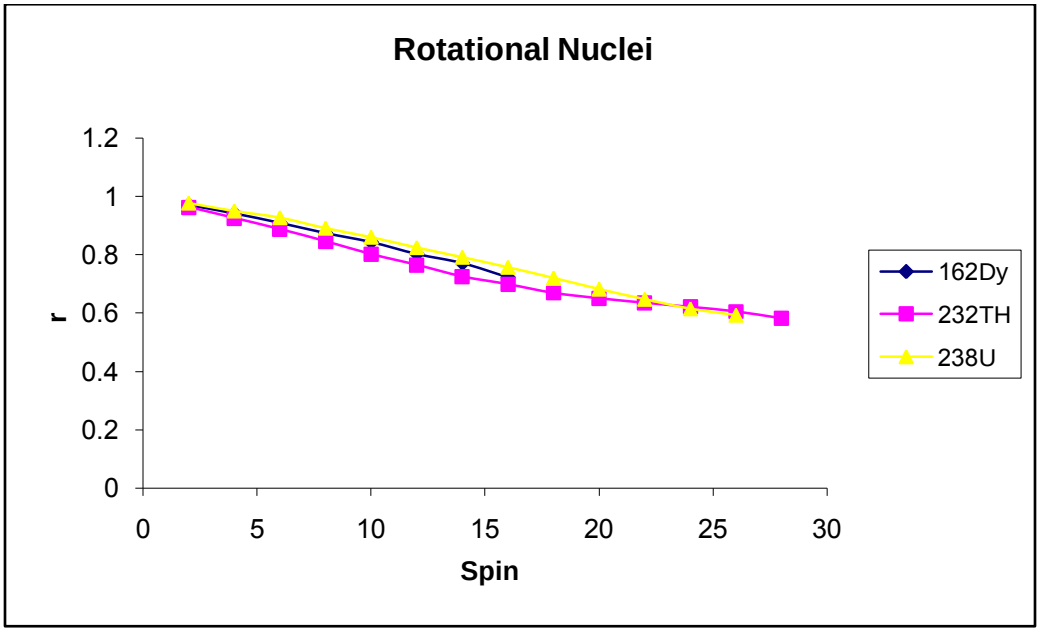

Fig. (1): The variation of relative ratio with spin.

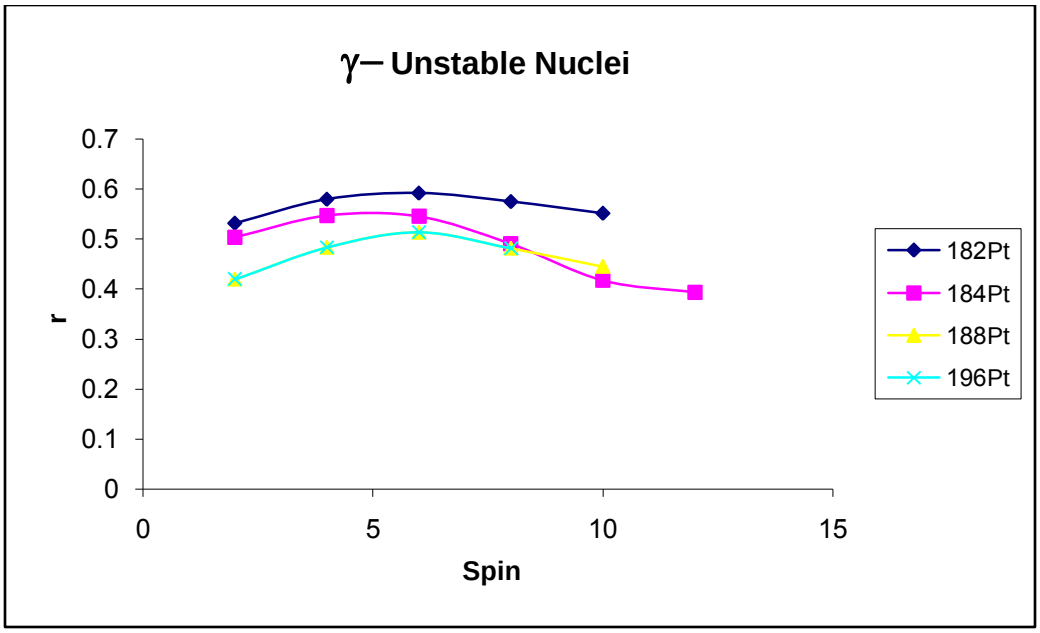

Fig. (2): The variation of relative ratio with spin.

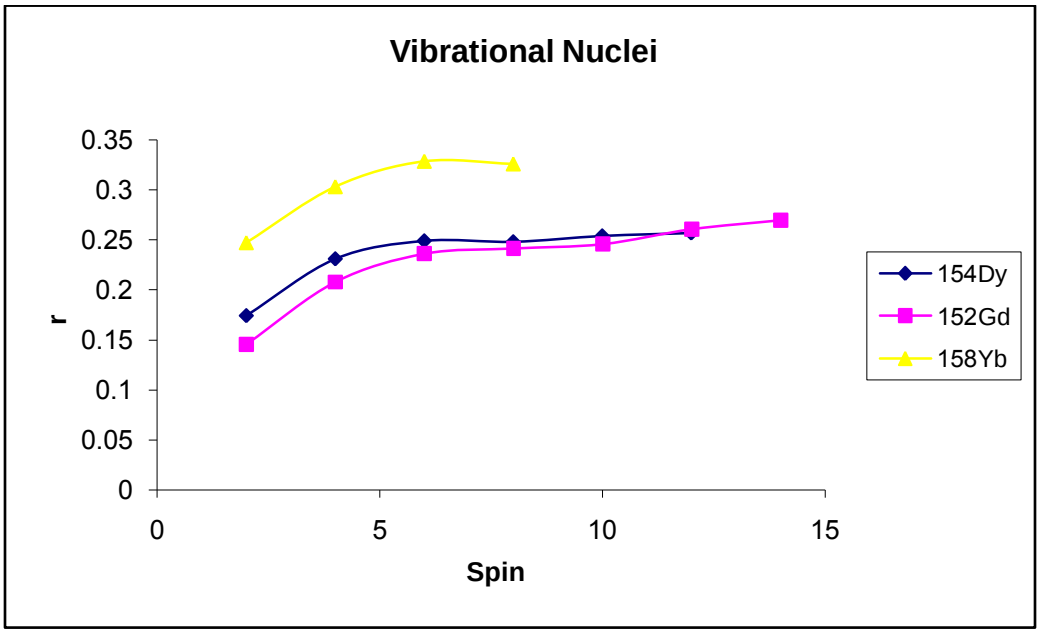

Fig. (3): The variation of relative ratio with spin.

LIMITATIONS OF STUDY:

This study is a theoretical adaptation of VMI and B. P. expressions equation to predict E value in terms of ground state moment of inertia (θ_0) parameter and stiffness parameter (C) for super deformed bands. The results were compared with experimental values provided in ENSDAT database. A software program was generated for computation and calculations. Therefore this model requires validation through experimental confirmation

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